$$E(r,z) \propto e^{-r^2/w(z)} \, e^{-jkr^2/2R(z)} \, e^{-j\tan^{-1} z/z_0}$$

$$\frac{1}{q} = \frac{1}{R(z)} - j\frac{\lambda}{\pi W^2(z)}$$

$$q_2 = \frac{A q_1 + B}{C q_1 + D}$$ ABCD Law Gaussian beams

$$O \xrightarrow{\ell} \qquad q_1 = z_{01} + jz$$

$$\begin{bmatrix} 1 & \ell \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1/f & 1 \end{bmatrix} = \begin{bmatrix} 1-\ell/f & \ell \\ -1/f & 1 \end{bmatrix}$$

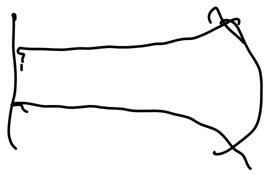
$$\frac{1}{q_2} = \frac{-1/f + 1/q_1}{1 - \ell/f + \ell/q_1} \rightarrow \frac{\lambda}{\pi W_{02}^2} = \frac{1/z_{01}}{(1-\ell/f)^2 + (\ell/z_0)^2}$$

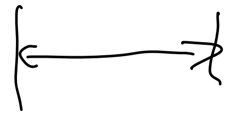
$$q = \frac{Aq+B}{Cq+D}$$

$$\frac{1}{q} = \frac{C + D/q}{A + B/q} \Rightarrow \frac{A}{q} + \frac{B}{q} = C + \frac{D}{q}$$

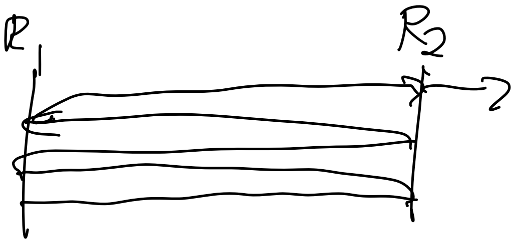
$$\frac{1}{q} = -\frac{(A-D)}{2B} \pm j\frac{\sqrt{1 - \left(\frac{A+D}{2}\right)^2}}{B}$$

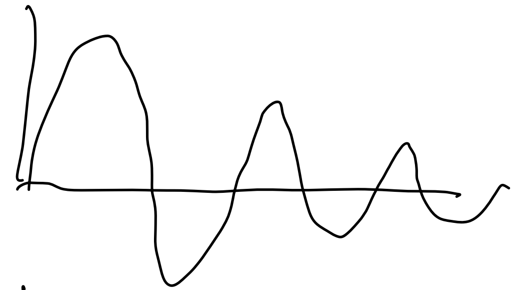
$$= \frac{1}{R(z)} - j\frac{\lambda}{\pi W_0^2(z)}$$

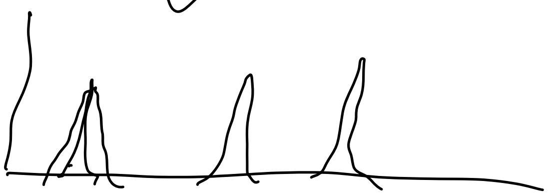




$$\frac{c}{2dn} = FSR$$







$$E_0 t_1 e^{ikd} t_2 +$$

$$E t_1 t_2 r_1 r_2 \cdots$$

$$E_T = E_0 t_1 t_2 \left[1 + r_1 r_2 e^{i2kd} + r_1^2 r_2^2 e^{i4kd} + \dots \right]$$

$$\frac{I}{I_0} = \frac{(1-R_1)(1-R_2)}{(1-\sqrt{R_1 R_2})^2 + 4\sqrt{R_1 R_2}\sin\theta}$$

$$\theta = \frac{2\pi\nu d}{c}$$

$$(1-\sqrt{R_1 R_2})^2 = 4\sqrt{R_1 R_2}\sin^2\theta$$

$$\Delta\nu = \frac{c}{2d}\left(\frac{1-\sqrt{R_1 R_2}}{\pi\sqrt{R_1 R_2}}\right)$$

$$Q = \frac{\nu}{\quad} = \frac{2\pi d}{\quad} \frac{(R_1 R_2)^{1/4}}{\quad (R_1 R_2)^{1/2}}$$

$\tau \quad \Delta\nu \qquad \lambda \qquad 1-(R_1R_2)^{1/4}$

finesse

$$\frac{FSR}{\Delta\nu} = \frac{\pi (R_1R_2)^{1/4}}{1-(R_1R_2)^{1/2}}$$

energy decay time
photon lifetime

$$N \rightarrow R_1N \rightarrow R_1R_2N$$

$$(1-R_1R_2)N$$

$$\frac{\Delta N}{\Delta t} = \frac{(1-R_1R_2)N}{2d/c} \rightarrow N(t) = e^{-t/\tau}$$

$$\tau = \frac{2d}{c}\left(\frac{1}{1-R_1R_2}\right)$$

Gain

start w/ blackbody radiation

$$\rho(\nu)\,d\nu = \frac{8\pi h\nu^3}{c^3}\,\frac{1}{e^{h\nu/kT}-1}$$

$$n\frac{\lambda_x}{2} = a \qquad k = \frac{2\pi}{\lambda}$$

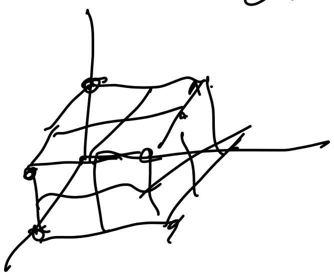
$$m\frac{\lambda_y}{2} = b$$

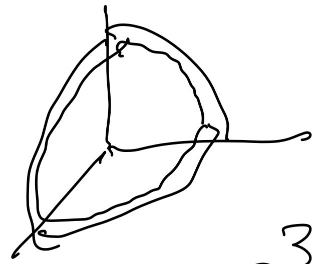
$$p\frac{\lambda_z}{2} = c$$

$$k^2 = k_x^2 + k_y^2 + k_z^2 \qquad k = \frac{2\pi n}{c}\nu$$

$$\nu^2 = \left(\frac{c}{\quad}\right)^2(n^2+m^2+p^2)$$

$(2na)$





$$V = \frac{1}{8}\frac{4\pi R^3}{3} \qquad R = \frac{2na\nu}{c}$$

\# of modes $$N = \underbrace{2}_{\text{polarization}} \times \frac{1}{8}\frac{4\pi}{3}\left(\frac{2a\nu}{c}\right)^3$$

$$\rho(\nu) = \frac{1}{V}\frac{dN}{d\nu}$$

$$\hookrightarrow \rho(\nu) = \frac{8\pi n^3}{c^3}\nu^2$$

$$\rho(\nu)\,d\nu = \frac{8\pi n^3}{c^3}\nu^2 \cdot kT \cdot d\nu$$

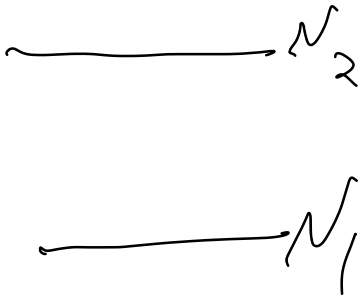
Rayleigh-Jeans Law

$E = nh\nu$

Bose-Einstein Distribution

$$\langle E_\nu \rangle = \frac{\sum nh\nu\, e^{-\frac{nh\nu}{kT}}}{\sum e^{-\frac{nh\nu}{kT}}} = h\nu \underbrace{\left(\frac{1}{e^{\frac{h\nu}{kT}} - 1}\right)}_{\#\text{ of photon}}$$

$$\rho(\nu) = \underbrace{\frac{8\pi n^3 \nu^2}{c^3}}_{\text{DOS}}\ \underbrace{h\nu}_{\text{energy}} \left(\frac{1}{\underbrace{e^{h\nu/kT} - 1}_{\text{how many photons per mode}}}\right)$$



spontaneous emission $\quad \dfrac{dN_2}{dt} = -A_{21} N_2$

absorption $\quad \dfrac{dN_2}{dt} = +B_{12} N_1 \rho(\nu) g(\nu)$

stimulated emission $\quad \dfrac{dN_2}{dt} = -B_{21} N_2 \rho(\nu) g(\nu)$

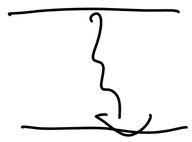
$$\frac{A_{21}}{B_{21}} = \frac{8\pi n^3 h \nu^3}{c^3}$$

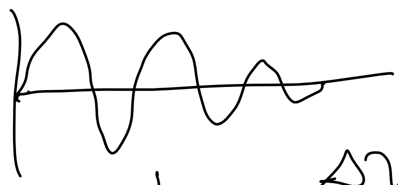
can measure A

$N_2 = N e^{-A_{21} t}$

Homogeneous broadening

Inhomogeneous broadening





homogeneous

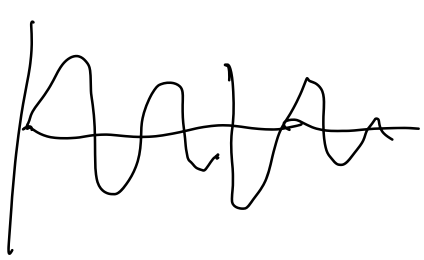
$$g(\nu) = \frac{1}{2\pi} \frac{\Delta\nu_{1/2}}{(\nu_0-\nu)^2 + \left(\frac{\Delta\nu}{2}\right)^2}$$

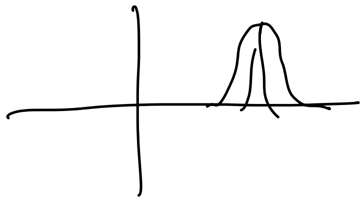
Doppler broadening - ihomogeneous

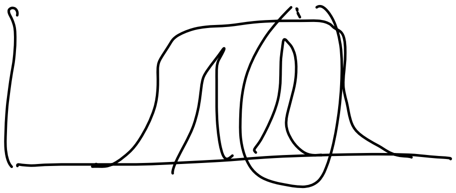
$$\nu_{obs} = \nu_0 \left[1 + \frac{v}{c} \right]$$

Gaussian

Pressure broadening – homogeneous

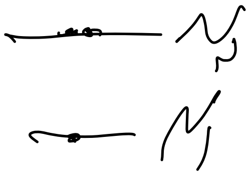






$$\frac{dN_2}{dt} = -A_{21}N_2 - B_{21}N_2\,\rho(\nu)g(\nu) + B_{21}N_1\,\rho(\nu)g(\nu)$$

$$\rho(\nu) = \frac{8\pi n^3 \nu^2 h\nu}{c^3}\,\frac{1}{e^{h\nu/kT}-1}$$



$$\rho(\nu) = \frac{A_{21}}{B_{21}}\,\frac{1}{\frac{B_{12}\,g_1}{B_{21}\,g_2}e^{h\nu/kT}-1}$$

$$\frac{dN_2}{dt} = -A_{21}N_2 - \underbrace{\frac{A_{21}\lambda^2}{8\pi n^2}\,g(\nu)}_{\text{cross-section } \sigma}\,\frac{I}{h\nu}\left(N_2 - \frac{g_2}{g_1}N_1\right)$$